SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P1
HIGHER GRADE
2014

MARKS: 200
TIME: 3 hours

This question paper consists of 9 pages, 1 answer sheet and 1 formula sheet.

X05
INSTRUCTIONS AND INFORMATION

Read the instructions below carefully before answering the questions.

1. This question paper consists of EIGHT questions. Answer ALL the questions.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

3. You may use an approved calculator (non-programmable and non-graphical), unless stated otherwise.

4. If necessary, round answers off to TWO decimal places, unless stated otherwise.

5. Use the ANSWER SHEET only for QUESTION 8. Write your centre number and examination number in the spaces on the ANSWER SHEET. Detach it and place it inside the front cover of the ANSWER BOOK.

6. Number the answers correctly according to the numbering system used in this question paper.

7. Diagrams are not necessarily drawn to scale.

8. A formula sheet is included at the end of the question paper.

9. Write neatly and legibly.
QUESTION 1

1.1 Solve for $x$:

1.1.1 $x - 5 = \frac{6}{x}$

1.1.2 $(2x + 3)(3 - x) = 2$ (Correct to TWO decimal places)

1.1.3 $4x - 3 = 4\sqrt{x}$

1.2 Determine the values of $x$ which will satisfy the following inequalities simultaneously:

$|2 - x| > \frac{11}{2}$ and $x^2 \leq 100$

1.3 Tiles are used to cover a rectangular area. There are 5 rows of tiles and each row contains 12 tiles. The covered area must be doubled by increasing the length and width by the same number of tiles. Determine the number of rows in the larger rectangle.

1.4 For which values of $k$ will the equation $(x - k)(2 - x) = 9$ have real roots?

1.5 Prove that the roots of the equation $x^2 + (2 - p)x - 3 - p = 0$ are real and unequal for all real values of $p$. 

[39]
QUESTION 2

2.1 The sketch graphs represent the functions \( g(x) = |x-p| \) and \( f(x) = ax^2 + bx \). A(2 ; 0) and B(0 ; 2) are intercepts of \( g \) with the axes and \( R \) is the reflection of \( B \) in the line \( x = 2 \). The graph of \( f \) contains \((0 ; 0), A \) and \( R \).

2.1.1 Determine the value of \( p \). \hspace{1cm} (2)

2.1.2 Write down the coordinates of \( R \). \hspace{1cm} (1)

2.1.3 Hence show that \( a = \frac{1}{4} \) and \( b = -\frac{1}{2} \). \hspace{1cm} (4)

2.1.4 Determine the coordinates of the turning point \( T \) of \( f \). \hspace{1cm} (3)

2.1.5 K is any point on the graph of \( g \). If \( KM \) is parallel to the \( y \)-axis with \( M \) on the graph of \( f \), determine the largest possible value of \( KM \). \hspace{1cm} (6)
2.2 The figure below shows the graphs of \( g(x) = a^x \) and \( h(x) = \frac{k}{x} \) \((x < 0)\). The point \( A(-1; 3) \) lies on both \( g \) and \( h \) and the point \( B \) is the \( y \)-intercept of \( g \).

2.2.1 Determine the values of \( a \) and \( k \). \( (3) \)

2.2.2 Write down the coordinates of \( B \). \( (1) \)

2.2.3 Determine the equation of \( g^{-1} \), the inverse of \( g \), in the form \( g^{-1}(x) = \ldots \) \( (2) \)

2.2.4 Calculate the values of \( x \) for which \( g^{-1}(x) > -1 \). \( (3) \)

**QUESTION 3**

3.1 State the factor theorem without proof. \( (2) \)

3.2 If \( f(x) = -x^2 - 4x + 5 \) is a factor of \( g(x) = x^3 - px^2 + mx + 10 \), determine the numerical values of \( p \) and \( m \). \( (7) \)
QUESTION 4

4.1 If \( \log 2 = x \) and \( \log 3 = y \), write \( \log_{4} 36 \) in terms of \( x \) and \( y \). (4)

4.2 Without the use of a calculator, prove that \( \sqrt[3]{3} > \sqrt[6]{7} \). (2)

4.3 Simplify: \( \frac{3^x (3^{x-1})^{y+1}}{9^{x-1}} \) (4)

4.4 Solve for \( x \):

4.4.1 \( 4^x + 2^x = 8(2^x + 1) \) (6)

4.4.2 \( \log x - \frac{3}{\log x} = 2 \) (5)

4.4.3 \( \log_{\frac{1}{3}} (x+4) > -2 \) (6)

4.5 If \( \log(a-b) = \log a - \log b \), show that \( a = \frac{b^2}{b-1} \).

(Assume that \( a > b > 0 \) and \( b \neq 1 \).) (4) [31]

QUESTION 5

5.1 The fourth term of an arithmetic sequence is 13 and the ninth term is 33.

Determine:

5.1.1 The seventh term of the sequence (4)

5.1.2 The sum of the first 40 terms of the sequence (2)

5.2 Two arithmetic sequences, \( A \) and \( B \), have the same common difference. The first term of \( B \) is 5 more than the first term of \( A \). The sum of the first \( n \) terms of each sequence is calculated.

By how much is the sum of \( B \) larger than the sum of \( A \)? (5)

5.3 Given: \( \sum_{n=1}^{\infty} (-1)^{n+1} (x-2)^n \)

5.3.1 Determine the values of \( x \) for which the above series converges. (4)

5.3.2 Determine the sum to infinity of the series if \( x = 2 \frac{3}{4} \). (3)
5.4 A moving car slows down to a standstill. In the first second it moves 25 m, in the second second 20 m and in the third second 16 m. The sequence formed by these distances is a geometric sequence.

5.4.1 Calculate how many seconds the car takes to move \( \frac{29}{1+25} \) m.

5.4.2 Calculate the total distance the car covers before it comes to a standstill.

QUESTION 6

6.1 Given: \( g(x) = x^2 + 2x \)
Determine \( g'(x) \) from \textbf{first principles}.

6.2 Determine \( \frac{dy}{dx} \) if:

6.2.1 \( y = \frac{x - 3\sqrt{x}}{x^2} \)

6.2.2 \( \frac{y}{3x} = (1 + x)^2 \)

6.3 A function \( h \), given by \( h(x) = ax^2 + \frac{b}{x} \), has a minimum value of 12 if \( x = 2 \). Calculate the values of \( a \) and \( b \).
QUESTION 7

7.1 Consider the function \( f(x) = x^3 - 8x^2 + 5x + 14 \)

7.1.1 Determine the coordinates of the intercepts of \( f \) with the axes. (5)

7.1.2 Determine the coordinates of the turning points of \( f \). (6)

7.1.3 Hence draw a neat sketch of the graph of \( f \), clearly indicating the turning points and the intercepts with the axes. (5)

7.1.4 Use your graph to write down the values of \( t \) for which the equation \( x^3 - 8x^2 + 5x + 14 = t \) has one real root. (2)

7.1.5 Determine the equation of the tangent to the curve of \( f \) at \( x = 1 \). (5)

7.2 A rectangular box which is open at the top, has a square base with sides of \( x \) cm and a height of \( h \) cm.

7.2.1 Write down the equation for the volume \( V \) in terms of \( x \) and \( h \). (1)

7.2.2 If the volume is 256 cm\(^3\), find \( h \) in terms of \( x \). (1)

7.2.3 Write down an equation for the external surface area \( A \) in terms of \( x \). (3)

7.2.4 An open rectangular box is to be constructed so that it can hold 256 cm\(^3\) when completely full. What must its dimensions be so that as little material as possible is used in its construction? (5)
QUESTION 8

A businessman manufactures two products, X and Y. Denote by $x$ and $y$, the number of units he makes of X and Y respectively. The shaded area below represents the feasible region with respect to a set of constraints. The vertices of the polygon, namely P(15 ; 20), Q(15 ; 15), R(15 ; 5), S(40 ; 5), T(40 ; 10) and V(20 ; 20) are given. PV and RS are parallel to the x-axis while PQ and TS are parallel to the y-axis.

8.1 Write down ALL these inequalities. (8)

8.2 If the businessman makes a profit of R100 on each unit of product X and R40 on each unit of product Y, write down an equation in terms of $x$ and $y$ which will represent the profit (P) that the businessman makes. (2)

8.3 On the ANSWER SHEET, show the point $(x ; y)$ at which the profit is a maximum, by drawing the search line. (2)

8.4 What is the maximum profit that the businessman can make? (2)

8.5 What is the minimum profit that the businessman can make, subject to the same constraints, and how many units of each product should be manufactured? (3)

TOTAL: 200
ANSWER SHEET

QUESTION 8

This ANSWER SHEET should be handed in.
Mathematics Formula Sheet (HG and SG)
Wiskunde Formuleblad (HG en SG)

**Formulae**

- Quadratic formula: 
  
  \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

- Sum of first \(n\) terms of an arithmetic sequence:
  
  \[ S_n = \frac{n}{2} (a + T_n) \quad \text{or} \quad S_n = \frac{n}{2} (a + \ell) \]

- Sum of first \(n\) terms of a geometric sequence:
  
  \[ S_n = \frac{n}{2} [2a + (n-1)d] \]

- Sum to infinity of a geometric sequence:
  
  \[ S_\infty = \frac{a}{1 - r} \quad (|r| < 1) \]

- Area of a triangle:
  
  \[ A = P \left( 1 + \frac{r}{100} \right)^n \quad \text{or} \quad A = P \left( 1 - \frac{r}{100} \right)^n \]

- Derivative of a function:
  
  \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

- Distance formula:
  
  \[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

- Equation of a line:
  
  \[ y = mx + c \]

- Distance between two points:
  
  \[ (x_3,y_3) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

- Pythagorean theorem:
  
  \[ x^2 + y^2 = r^2 \]

- Equation of a circle:
  
  \[ (x - p)^2 + (y - q)^2 = r^2 \]

- In \(\Delta ABC\):
  
  \[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \]

  \[ a^2 = b^2 + c^2 - 2bc \cdot \cos A \]

  \[ \text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C \]